Name: Computer number: Serial numper

Math 312 Second Exam 17- 6- 1434

Prove the following:

1. A subset M of a complete metric space X is itself complete if and only if the set M is closed in X. 5

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a. Suppose $f : [a, b] \to [a, b]$ is continuous then f has a fixed point in $[a, b]$. 2.5

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b. Under what condition f has a unique fixed point. 0.5

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2. Let Ω be the set of all invertible linear operators on $\mathbb{R}^n$, then if $A \in \Omega$ and $B \in L(\mathbb{R}^n)$ such that $\| B - A \| \| A^{-1} \| < 1$, then $B \in \Omega$. 4

a. Define the continuously differentiable mapping. 1

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b. Suppose f maps an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$. If f is continuously differentiable on E, then the partial derivatives $D_j f_i$ is continuous. 3

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3. $\mathbb{R}^n$ is complete metric space. 4